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XLIII. *A Demonstration of Two Theorems mentioned in Art. XXV. of the Philosophical Transactions for the Year 1775. In a Letter from Charles Hutton, Esq. F. R. S. to the Rev. Dr. Horsley, Sec. R. S.*

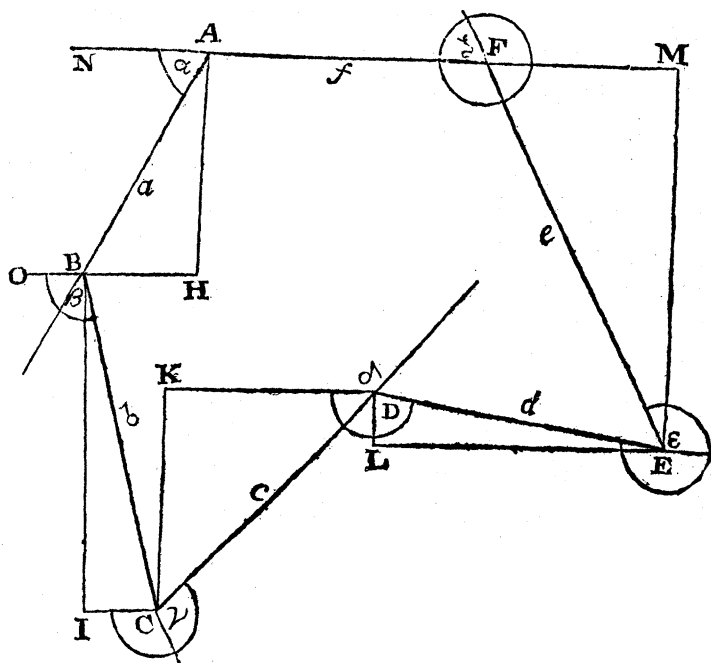
TO THE REV. DR. HORSLEY, SEC. R. S.

REV. SIR,

Woolwich Warren, Royal Mil. Acad.
April 17, 1776.

R. June 27, 1776. **T**HE following is a demonstration of the two theorems mentioned in Art. XXV. of the Philosophical Transactions for the year 1776. If you think them proper to be inserted in the next volume of Transactions, be pleased to communicate them to the Royal Society for that purpose, from, &c.

THE



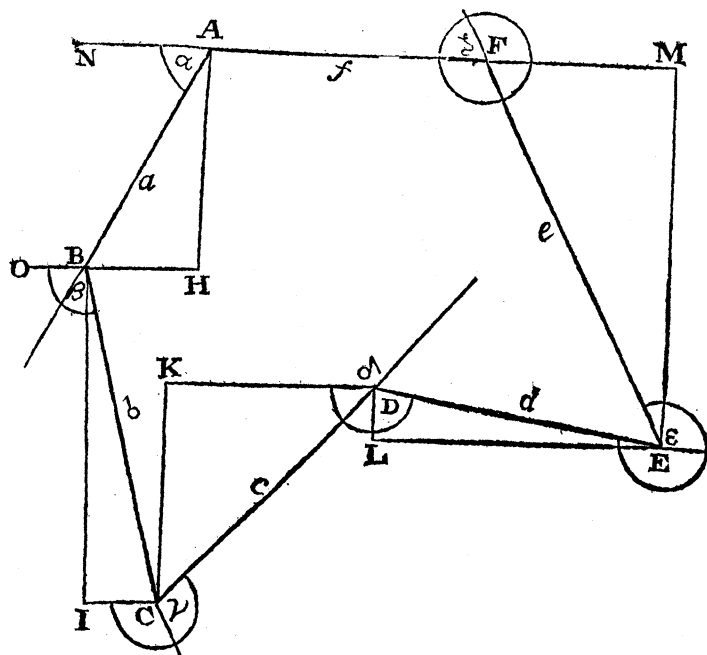
THE DEMONSTRATION.

Parallel to one side, AF, of the polygon ABCDEF,, through all the angular points, draw BH, CI, DK, EL, FM. Then are the several angles, which are made by these parallels and the adjacent sides of the figure, respectively equal to the sums of all the exterior angles of the figure, from the beginning to the place of each angle; by the beginning is meant either extremity of the side AF. Thus the $\angle NAB = \alpha$, the $\angle OBC = \alpha + \beta$, the $\angle ICD = \alpha + \beta + \gamma$, the $\angle KDE = \alpha + \beta + \gamma - \delta$ (for here δ , being without the figure, is subtracted instead of being added)

added), the $\angle LEF = \alpha + \beta + \gamma - \delta + \varepsilon$, &c. All this is evident from Eucl: I. 29.; the angles we speak of being those, which are measured by the little arcs described about each angular point in the figure.

I. If right lines, AH, BI, CK, DL, EM, be drawn from the angular points, perpendicular each to the parallel which passes through the next angular point, the sums of the perpendiculars, drawn in contrary directions, the one upwards, and the other downwards, will be equal. And each perpendicular will be a fourth in proportion with the radius, that side of the polygon which is adjacent both to the perpendicular and the parallel on which it falls, and the sine of the sum of the external angles taken to that inclusively from which the perpendicular is drawn. Thus, $AH + BI + DL = CK + EM$; and $\text{rad. fin. } \alpha = AB : AH$; and $\text{rad. fin. } \alpha + \beta = BC : BI$, and in like manner of the rest. Take the value, therefore, of each perpendicular by these analogies, putting unity for the radius, subtract the sum of all that are drawn upwards from the sum of all that are drawn downwards, and the remainder, put equal to 0, is the first equation; that is, $AH = a \times \text{f. } \alpha$ (for $\angle ABH = \angle \alpha$); $BI = b \times \text{f. } \alpha + \beta$ (for $\text{f. } \angle ICB = \text{f. } \angle HBC = \text{f. of its suppl. } OBC \text{ or } \alpha + \beta$); in like manner, $CK = -c \times \text{f. } \alpha + \beta + \gamma$; $DL = d \times \text{f. } \alpha + \beta + \gamma - \delta$; $EM = -e \times \text{f. } \alpha + \beta + \gamma - \delta + \varepsilon$, &c. the last perpendicular will always be = 0, because the sine of 360° , or of $\alpha + \beta + \gamma - \delta + \varepsilon + \zeta$ is nothing. Hence $a \times \text{f. } \alpha + b \times \text{f. } \alpha + \beta + c \times \text{f. } \alpha + \beta + \gamma + d \times \text{f. } \alpha + \beta + \gamma - \delta + e \times \text{f. } \alpha + \beta + \gamma - \delta + \varepsilon = AH + BI - CK + DL - EM = 0$, which is the first equation.

II.



II. In like manner it appears, that the intercepted parts BH, CI, DK, EL, FM, AF, of the parallels before-mentioned, are equal to the several corresponding sides drawn into the cosines of the same sums of the exterior angles (the radius being 1.); and because $BH - CI - DK - EL + FM + AF = 0$, therefore $a \times \cos. \alpha + b \times \cos. \alpha + \beta + c \times \cos. \alpha + \beta + \gamma + d \times \cos. \alpha + \beta + \gamma - \delta + e \times \cos. \alpha + \beta + \gamma - \delta + \varepsilon + f \times \cos. \alpha + \beta + \gamma - \delta + \varepsilon + \zeta = 0$, which is the second theorem. Or, for the last term, $f \times \cos. \alpha + \beta + \gamma - \delta + \varepsilon + \zeta$, of this latter theorem, might be substituted its value f only.